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Analysis of an N-Port Consisting of a Radial Cavity and E-Plane Coupled Rectangular Waveguides

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Abstract—**An analysis of an n-port including a radial or coaxial cavity and E-plane coupled rectangular waveguides is presented. A non-standard field matching technique which exploits both circular and rectangular boundaries, is used to determine the scattering matrix parameters of the n-port. Validity of the analysis is verified through comparison with an alternative analysis and experiment.**

Introduction

Microwave networks consisting of a number of rectangular waveguides coupled in the E-plane to a radial or coaxial cavity find many useful applications in microwave engineering. A well known rat-race circuit consisting of four waveguides coupled to a coaxial cavity is a typical example.

Recently, some interest has been shown in E-plane coupled waveguide five ports [1]–[4]. It has been demonstrated that these five-ports can be used as power combiners [4] or as building blocks for six-port network analysers [1], [3].

So far, the design of E-plane coupled waveguide n-ports has been based on experiment (i.e. [1], [4]). The only theoretical analysis of an E-plane waveguide n-port has been presented in [2]. The analysis was restricted to the symmetrical five-port junction and was based on the least-squares boundary residual method (LSBRM). A good agreement with experiment was noted.

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In order to set equations for unknown modal expansion coefficients the authors in [1] used the continuity conditions for tangential components of the fields only along the circular contour of the cavity. Since equations for unknown expansion coefficients for the cavity region were inseparable from those of the rectangular waveguide region, the size of the resulting matrix was large.

In this paper an alternative analysis based on a field matching technique for an E-plane n-port is described. In difference to the method presented in [1] both rectangular and circular boundaries are exploited in setting up the continuity conditions for the tangential field components. This approach leads to separation of modal expansion coefficients for the waveguide region from those of the cavity region. In this way a considerable reduction of the size of the matrix involved in solving equations for unknown modal expansion coefficients is achieved.

The solution presented here exhibits good convergence and is easily implemented on an IBM PC or compatible.

Analysis

The structure of the analyzed n-port circuit is shown in Fig. 1.

The n-port consists of N radially positioned rectangular waveguides which are connected in the E-plane to a radial or coaxial cavity. The positions of the rectangular guides with respect to the radial or coaxial cavity are given by angles Φ_i, $i = 1, \cdots, N$.

From the designers point of view, the parameters of interest are the scattering matrix coefficients of the n-port.

Assuming dominant mode operation of the individual waveguides, the determination of the scattering parameters requires the n-times solving of an electromagnetic problem, in which one of the waveguides is connected to the generator and the remaining waveguides are match-terminated. There are a number of ways to solve this formulated EM problem. The method chosen here is based on the non-standard field matching technique which exploits both circular and rectangular natural boundaries, associated with the geometry of the N-port.

Field Matching Solution

It is sufficient to present the method for the case when waveguide No. 1 is excited and the remaining waveguides are match-terminated.

Under the condition of the dominant mode operation, the rectangular waveguides support free propagation of the TE_{10} mode. The other modes are excited at the inter-junctions between rectangular waveguides and the cavity, but quickly decay over distance. Due to the form of excitation the waveguide and cavity modes combine in such a way that the y-component of the electric field is zero. In this case the total field in all the regions of the n-port can be considered as the radial TE. Its components can be derived from the knowledge of the y-component of the magnetic field.

The y-component of the magnetic field in the waveguides can be written in the Cartesian system of coordinates in the form (1):

$$H_y^I = \frac{-j\Gamma_1^2}{kZ}\left[\frac{e^{-\Gamma_{01}z}}{\Gamma_{01}}\delta_{1i} - \sum_{m=0}^{\infty} A_{im}\cos(k_{xm}x)\frac{e^{\Gamma_{m1}z}}{\Gamma_{m1}}\right]\sin(k_{y1}y) \tag{1}$$

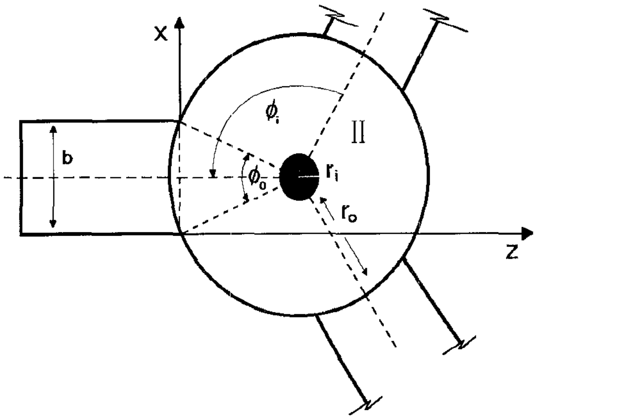

Fig. 1. N-port circuit consisting of a radial or coaxial cavity and E-plane coupled rectangular waveguides.

where:

k is a wave number,
Z is an intrinsic impedance,
δ_{1i} is the Kronecker's delta.

$$k_{xm} = \frac{m\pi}{b}, \quad k_{y1} = \frac{\pi}{a} \text{ are eigenvalues, } \Gamma_1^2 = k^2 - k_{y1}^2$$

$$\Gamma_{ml}^2 = k_{xm}^2 + k_{y1}^2 - k^2, \quad j = \sqrt{-1},$$

A_{im} – are expansion coefficients.

and x, y, z are Cartesian coordinates which are associated with the ith waveguide.

Note that in expression (1) coefficient $-j\Gamma_1^2/kZ$ was introduced in order to simplify expressions for the associated electric fields which are derived later in (5).

Also note that $\Gamma_1(\Gamma_1^2 = -\Gamma_{01}^2)$ is used in preference to Γ_{01} in expressions (3), (6) derived for the fields in the cavity region.

In expression (1), coefficient A_{10} is the reflection coefficient or S_{11} parameter of the scattering matrix. Coefficients $A_{io}(i \neq 1)$ are transmission coefficients or S_{i1} elements of the scattering matrix.

The representation in (1) is inconvenient to use at the circular inter-junctions. Thus the representation in (1) is replaced by its radial co-ordinate equivalent shown by (2):

$$H_y^I = \frac{-j\Gamma_1^2}{kZ}\left[\frac{e^{\Gamma_{01}(r\cos\acute{\Phi} - z_o)}}{\Gamma_{01}}\delta_{1i} - \sum_{m=0}^{\infty} A_{im}\cos k_{xm}\left(r\sin\acute{\Phi} + \frac{b}{2}\right) \cdot \frac{e^{-\Gamma_{m1}(r\cos\acute{\Phi} - z_o)}}{\Gamma_{m1}}\right]\sin(k_{y1}y) \tag{2}$$

where:

$$\acute{\Phi} = \Phi - \Phi_i \quad \text{for the } i\text{th part}$$

$$x = \frac{b}{2} + r\sin\acute{\Phi}, \quad z = z_o - r\cos\acute{\Phi} \quad \text{and } z_o = r_o\cos(\Phi_o/2)$$

For the cavity region the y-component of the magnetic field can be expressed in the form (3):

$$Hy^{II} = \frac{-j\Gamma_1}{kZ}\sum_{l=-\infty}^{\infty} F_l e^{jl\Phi} R_l(r)\sin(k_{y1}y) \tag{3}$$

where: F_l are expansion coefficients and functions R_l are defined by

$$R_l(r) = \begin{cases} \dfrac{J_l(\Gamma_1 r)}{J_l'(\Gamma_1 r_o)} & \text{for a radial cavity} \\[2ex] \dfrac{J_l(\Gamma_1 r)Y_l'(\Gamma_1 r_i) - Y_l(\Gamma_1 r)J_l'(\Gamma_1 r_i)}{J_l'(\Gamma_1 r_0)Y'l(\Gamma_1 r_i) - Y_l'(\Gamma_1 r_o)J_l'(\Gamma_1 r_i)} & \text{for a coaxial cavity} \end{cases}$$

J_l, Y_l are Bessel and Neumann functions respectively, symbol "'"-indicates derivatives.

It can be shown that, in order to determine the scattering parameters of the n-port, it is sufficient to know only the y-component of the magnetic field and the Φ-component of the electric field.

For the radial TE field having variation sin $(k_{y1}y)$ in the y variable, the following relationship between the y-component of the magnetic field and the $\emptyset$-component of the electric field can be exploited (4):

$$E_\Phi = \frac{jkZ}{\Gamma_1^2}\frac{\partial H_y}{\partial r} \tag{4}$$

By using (4) the Φ-component of the electric field in the waveguides regions can be found to be given by (5):

$$E_\Phi^I = \left\{\cos\acute{\Phi}\, e^{\Gamma_{01}(r\cos\acute{\Phi} - Z_o)}\delta_{1i} + \sum_{m=0}^{\infty} A_{im}\left[\frac{k_{xm}}{\Gamma_{m1}}\sin\acute{\Phi}\sin k_{xm} \cdot \left(r\sin\acute{\Phi} + \frac{b}{2}\right) + \cos k_{xm}\left(r\sin\acute{\Phi} + \frac{b}{2}\right)\cos\acute{\Phi}\right] \cdot e^{-\Gamma_{m1}(r\cos\acute{\Phi} - Z_o)}\right\}\sin k_{y1}y \tag{5}$$

For the cavity region, the $\emptyset$-component of the electric field is given by (6):

$$E_\Phi^{II} = \sum_{l=-\infty}^{\infty} F_l e^{jl\Phi} R_l'(r)\sin(k_{y1}y) \tag{6}$$

In order to determine the unknown expansion coefficients A_{im} and F_l, the continuity conditions for the y-component of the magnetic field and the Φ-component of the electric field can be used. For mathematical convenience, the required continuity conditions can be set along the constant-coordinate (rectangular or circular) contours [1].

In [1] only a circular boundary was exploited. In this paper continuity conditions are applied along both circular and rectangular co-ordinates and the continuity equations are give by (7) and (8):

At $r = r_o$ (circular aperture):

$$E_\Phi^{II} = \begin{cases} E_\Phi^I & \text{for } \Phi_i - \Phi_o/2 \le \Phi \le \Phi_i + \Phi_o/2 \\ 0 & \text{elsewhere} \end{cases} \tag{7}$$

At $z' = 0$ (rectangular aperture):

$$H_y^I = H_y^{II} \quad \text{for } 0 \le x_i' \le b \qquad i = 1, \cdots, N \tag{8}$$

Equations (7) and (8) are in the functional form. In order to convert (7) and (8) to algebraic equations the Fourier analysis can be applied. By multiplying both sides of equation (7) by exp $(-jp\Phi)$ and by integrating within the limits 0 to 2π, coefficients F_l can be expressed in terms of coefficients A_{im}. Now, by using (7) coefficients F_l can be eliminated from (8).

Now to create the final set of equations for unknown coefficients A_{im}, a circular boundary (only in the aperture regions) as in [1] can be used. This choice however, creates difficulties. A new set (preferably orthonormal) of weighting functions over $\phi_i - \phi_o/2 < \phi < \phi_i + \phi_o/2$, needs to be created to convert functional equations into algebraic equations. The new set will not exhibit mutual orthogonality with functions describing the waveguide modes.

In this context, the rectangular contour and mutually orthogonal set of functions cos $(k_{xn}x')$ in (8) are the better choice.

By multiplying both sides of (8) by cos $(k_{xn}x')$ and by integrating within the limits $0 \leq x_i' \leq b$ a set of final equations for unknown expansion coefficients A_{im} can be obtained.

The set of algebraic equations for A_{im} can be easily solved by using the standard Gauss elimination method.

By solving for A_{im} the first column of the scattering matrix is determined.

In order to determine the remaining elements of the scattering matrix, the procedure demonstrated for port 1 has to be repeated for the remaining (n − 1) ports.

At this point a short comparison between the LSBRM and the current method, based on the field matching technique, can be made.

The difference between the LSRBM and the current method is quite clear. In the LSBRM method minimization of the residue involves solving simultaneous equations for A_{im} and F_l coefficients. In the analysis presented here, only equations for coefficients A_{im} need to be solved.

Computer Implementation

The fields in (2), (3) are given in terms of infinite expansions which have to be truncated for the computational purposes. If only one rectangular waveguide expansion mode (having dependence approximately constant with ϕ) is used, the number of radial harmonics in the cavity should be approximately taken as $2\pi r_o/b$. If more than one waveguide mode is used the number of radial harmonics must be proportionally increased.

The expressions for the field in the cavity (2) require the use of Bessel and/or Neumann functions. It should be noted that for orders 1 much greater than the values of arguments, Bessel and Neumann functions can be approximated by simple polynomials. More accurate values could be obtained by using backward recurrence formulae [1], [5]. The other approximations involved functions $R_l(r)$. For small deviations in radial direction function $R_l(r)$ can be approximated by $R_l(r_o) + \Gamma_1(r - r_o)$.

Equations for unknown coefficients A_{im} require evaluation of integrals which involve products of functions appearing in (1), (2) and exp $(-jp\Phi)$ or cos $(k_{xn}x')$. Not all of these integrals can be given in the closed-form and in such cases numerical integration is necessary.

Results

Based on the theory described above a computer algorithm for determining the scattering matrix coefficients for an n-port incorporating a radial or coaxial cavity and E-Plane coupled rectangular waveguides has been developed.

Because of its simplicity, the algorithm could be easily implemented in Microsoft Fortran for an IBM PC or compatible.

The developed algorithm could be verified for the E-plane five-port circuits since the experimentation data was readily available [1]-[4].

Fig. 2 shows comparison between theoretical and experimental results for the return loss at one port of the symmetrical E-plane five-port incorporating a radial cavity. In calculations one and two

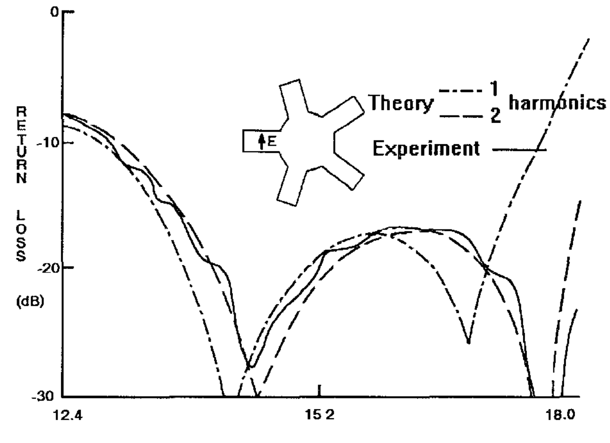

Fig. 2. Return loss versus frequency for a symmetrical 5-port consisting of a radial cavity and E-plane coupled rectangular waveguides. Cavity dimensions: height a = 15.8 mm, radius r_0 = 11.21 mm. Waveguide dimensions: width a = 15.8 mm, height b = 7.9 mm. Experimental points taken from Riblet [1].

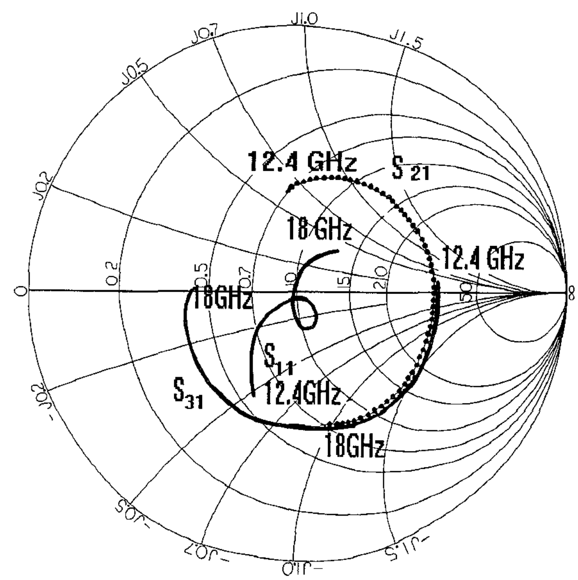

Fig. 3. Theoretical results for the scattering parameters of a symmetrical 5-port consisting of a radial cavity and E-plane coupled rectangular waveguides. 5-port dimensions are the same as in Fig. 2.

waveguide spatial harmonics (corresponding to m = 0, 1) and twenty radial cavity harmonics (l = 0, 1, · · · , 20) were used.

It can be noticed that the theoretical results obtained with one waveguide harmonic only roughly approximate the experimental results.

In order to investigate the convergence, calculations with four waveguide harmonics were also performed.

The results for the return loss obtained with four waveguide harmonics were virtually the same as those obtained with two harmonics. However, a slight difference was noted in the results for the phase of the reflection coefficient. Therefore, in all the further presented results, four waveguide spatial harmonics were used.

Fig. 3 shows the full set of scattering parameters for the same five-port, now presented in the Smith-chart format. Values of the scattering parameters indicate that in the 6-GHz wide frequency

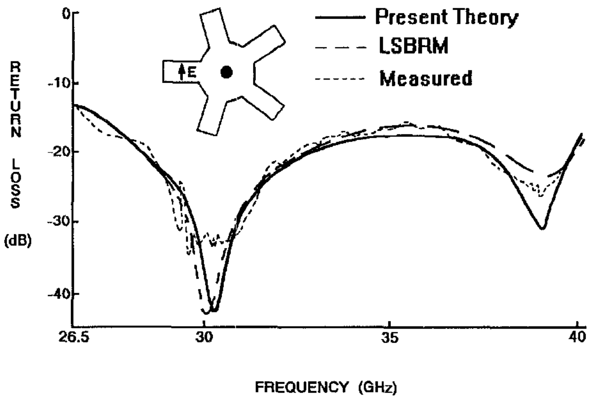

Fig. 4. Return loss versus frequency for a symmetrical 5-port consisting of a coaxial cavity and E-plane coupled rectangular waveguides. Cavity dimensions: height $a = 11.0$ mm, inner radius $r_i = 0.50$ mm, outer radius $r_o = 4.50$ mm. Waveguide dimensions: width $a = 11.0$ mm, height $b = 3.56$ mm. Experimental and LSBRM points taken from Cullen *et al.* [3].

band, the power delivered to port 1 is almost equally divided between the remaining four ports. At the same time, the phase difference between the signals at ports 2 and 3 stays approximately constant and is close to 120 degrees. According to the theory [1], the analysed five-port is suitable in the design of a broadband six-port reflectometer.

Fig. 4 shows a comparison between theoretical and experimental results for the return loss of the symmetrical five-port, now incorporating a coaxial cavity. The theoretical results were obtained by using LSBRM [3] and the non-standard field matching technique developed here. It can be seen that both LSBRM and the field-matching technique provide very similar results and both agree well with the experiment.

Conclusions

Based on the non-standard field matching technique which exploits both, circular and rectangular, natural boundaries, the analysis of an n-port comprising a radial or coaxial cavity and E-plane coupled rectangular waveguides has been presented. A computer algorithm for determining the scattering matrix of the n-port has been developed.

The validity of the new analysis and the algorithm have been verified by comparing numerical results with experiment and the alternative analysis which was based on the least-squares boundary residual method.

The new analysis produces results in good agreement with those produced by the least-squares boundary residual method and experiment. In comparison with the LSRBM, the new analysis is less complicated.

The analysis is easily implemented on an IBM PC. The analysis and computer algorithm can be of help to the designers of E-plane n-port circuits.

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Experimental Study of Multihole Directional Couplers Providing a Rippled Response

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***Abstract*—The directivity versus frequency of a multihole waveguide coupler may be expressed as a polynomial whose complex roots, if placed off the unit circle, provide the opportunity of a rippled response. Two designs using seven equispaced holes, one each with 3.0 and 0.5 dB ripple, were implemented at *X*-band and experimental validation of the theory has been excellent.**

Introduction

In 1985, Orchard *et al.* [1] introduced an iterative design technique used to synthesize shaped beam patterns for antenna array applications, with a specified ripple tolerance in the shaped region and sidelobes in the unshaped region at individually specified heights. Experimental verification followed quickly. In 1990, this method was extended to the design of multihole directional couplers by Elliott and Kim [2] who showed, for two rectangular waveguides with a common narrow or broad wall, how to determine coupling values so that the directivity response in the passband would be at a controlled ripple level rather than lobed, as in a Chebyshev design. The purpose of the present study was to use their technique to design two distinctly different narrow wall couplers, undertake their construction, and attempt an experimental confirmation of the theory.

Synthesis

Reference [2] contains a full description of the design procedure, embodied in (21)–(24), which permits calculation of coupling coef-

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